Paillier/ZKP

Ballot encryption, ZKP and weighted tallying in the Paillier cryptosystem

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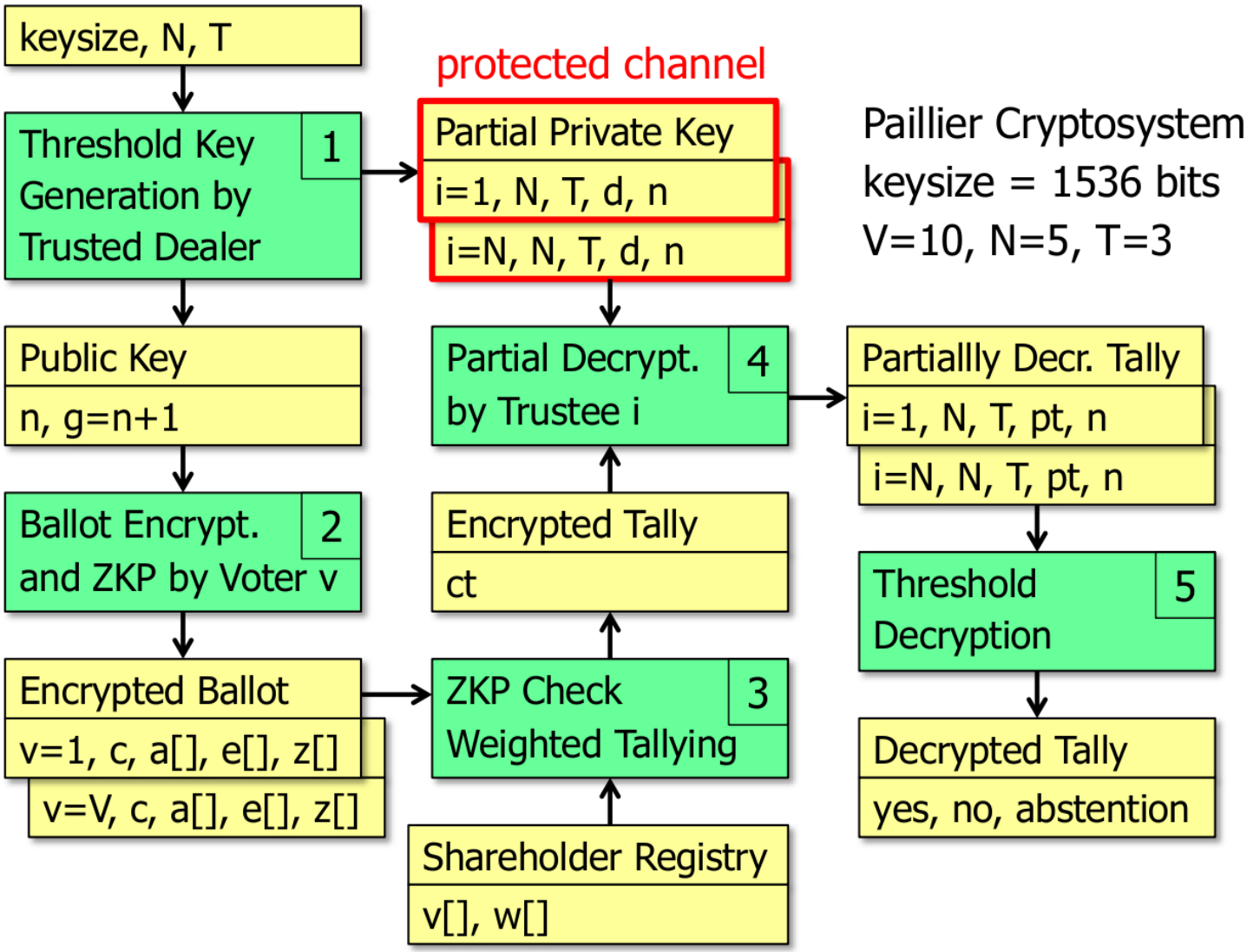
Outline

System Architecture



Our Modules (I)

Module 2

- Ballot encryption
- Zero-knowledge proof for ballot of voter v
- Input
  - Public key generated by module 1
  - Vote instruction (Candidate choice)
- Output
  - Encrypted vote (ballot)
  - ZKP for vote (a, e, z)
  - Election & voter id

Our Modules (II)

Module 3

- Zero-knowledge proof check of ballot
- Weighted tallying
- Input
  - Encrypted vote (ballot)
  - ZKP for vote (a, e, z)
  - Election & voter id
  - Voter registry (shares per voter)
- Output
  - Encrypted weighted tally (ct)
  - Election id

ZKP verification (I)

Prover must show that all u_k's are n^{th} powers

$$c = g^{m_i} \cdot r^n \ mod \ n^2 \tag{1}$$

$$u_k = c \cdot (g^{m_k})^{-1} \ mod \ n^2 \tag{2}$$

k	number of candidates
i	selected candidate
m_k	valid voting messages
u_k	bulletin board values

- Only possible for $k = i$
- Without disclosing random r!
- $\rightarrow$ Use a_k,e_k and z_k arrays to prove it

ZKP verification (II)

Sum of all e_k's must be equal to challenge e

$$e_k = \begin{cases} e - \sum_{k \neq i} e_k \ mod\ 2^b & k = i \\ e_k & k \neq i \end{cases} \tag{3}$$

$$\sum e_k \equiv e \ mod\ 2^b \tag{4}$$

$$b = \frac{size_2\,(n)}{2} \tag{5}$$

b	size of challenge in bits (768)
e	challenge (hashed voter/election data & commitment)
e_k	response array e

ZKP verification (III)

All z_k's must be n^{th} powers

$$a_k = \begin{cases} a_i = z_i^n \ mod \ n^2 & k = i \\ a_k = z_k^n \cdot \left(u_k^{e_k}\right)^{-1} \ mod \ n^2 & k \neq i \end{cases} \tag{6}$$

$$z_k = \begin{cases} z_i = z_i \cdot r^{e_i} \ mod \ n & k = i \\ z_k & k \neq i \end{cases} \tag{7}$$

$$z_k^n \equiv a_k \cdot u_k^{e_k} \ mod \ n^2 \tag{8}$$

a_k commitment

e_k response array e

z_k response array z

Weighted tallying

Encrypted tally is product of all encrypted votes modulo n^2

$$ct = \prod_{i=1}^{N_v} c_i^w \ mod \ n^2 \tag{9}$$

cw	weighted encrypted vote
w	weight (number of shares)
ct	encrypted tally
N_v	number of voters

Additive homomorphic properties

- $D\left(E\left(m_1\right) \cdot E\left(m_2\right) \ mod \ n^2\right) = m_1 + m_2 \ mod \ n$
- $D\left(E\left(m\right)^k \ mod \ n^2\right) = k \cdot m \ mod \ n$

Overview

Project information

- Programming Language: Ada
- Methodology: Test driven development (TDD)
- Coverage:
  - `-ftest-coverage -fprofile-arcs -fprofile-generate`
  - `lcov`
  - `genhtml`
- Dependencies:
  - Ahven (Unit test library)
  - GMP (Bignum)
  - GNATCOLL (JSON)

Project source

- Source code is freely available
- Opensource license: GPLv3+
- http://git.codelabs.ch/?p=paillier-zkp.git
- git clone http://git.codelabs.ch/git/paillier-zkp.git

Demo

Talk is cheap. Show me the code.
- Linus Torvalds

Questions

Thank you for your attention!